MA’RUZA

IKKINCHI TARTIBLI EGRI CHIZIQLAR. ELLIPS, GIPERBOLA, PARABOLA.

Ma’ruza rejasi

1. Ikkinchi tartibli egri chiziqlarning ta’rifi va ta’rifiy tenglamasini tuzish. A) Aylana B) Ellips C) Giperbola D) Parabola
2. Ular uchun umumiy bo’lgan markaz, ekssentrisitet tushunchalarini berish va ularning qiymatiga ko’ra shakl o’zgarishini ko’rsatish
3. II tartibli egri chiziqlarning umumiy tenglamasi va koeffisiyentlaridan tuzilgan aniqlovchilarning qiymatlariga ko’ra qaysi turga mansubligini ajratish (aniqlash)
4. Koordinatalarni almashtirish orqali ularning tenglamalarini kanonik shaklga keltirish.

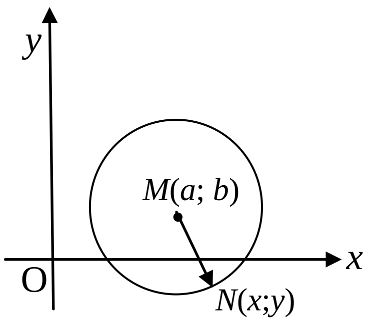
Tayanch so’z va iboralar: 2-tartibli egri chiziqlar, aylana, ellips, giperbola, parabola, aylana markazi, aylana radiusi, kanonik tenglamalar, fokuslar, direktrisa, ekssentrisitet, fokal radius, koordinatalarni almashtirish, koordinatalarni parallel ko’chirish, koordinatalar o’qini burish.

Avvalgi darsimizda to’g`ri chiziqning umumiy tenglamasini $Ax+By+C=0$ deb yozdik. Bunda nuqtaviy koordinatalarni ifodalovhi x, y kattaliklar birinchi darajada ishtirok etadi. Shu sababli bu tenglamani 1- tartibli (chiziqli) tenglama deb ataymiz .

$$Ax^2+2Bxy+Cy^2+Dx+Ey+F=0 \qquad (1)$$

tenglamada x^2, xy, y^2 ifodalar ikkinchi darajali ifodalardir. Shu sababli (1) tenglama ifodalovchi chiziqlarga ikkinchi tartibli egri chiziqlar deb aytamiz. Ikkinchi tartibli egri chiziqlarning eng sodda turlari: aylana, ellips,giperbola va parabolalardir.

I. <u>Aylana.</u> Markaz deb ataluvchi nuqtadan baravar uzoqlikda joylashgan nuqtalarning tekislikdagi geometrik o’rniga aylana deb aytamiz.Aytaylik, M(a,b) – markaz deb ataluvchi nuqta va N(x,y) -esa aylanaga taalluqli ixtiyoriy nuqta bo’lsin.



Shartga ko’ra, MN$=const=R$. Iikki nuqta orasidagi masofa formulasiga asosan:

$$R=MN=\sqrt{(x-a)^2+(y-b)^2}\Rightarrow$$

$$\Rightarrow (x-a)^2+(y-b)^2=R^2 \qquad (2)$$

Bu ,markazi M nuqtada bo’lib, radiusi R bo’lgan aylana tenglamasidir. Xususiy holda M(a; b)=O (0;0) bo’lsa, aylana tenglamasi

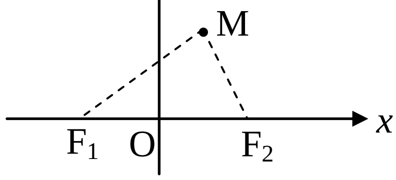
$$x^2+y^2=R^2 \quad (3)$$

bo’ladi. (2) va (3) tenglamalar aylananing kanonik (eng sodda) tenglamasi deb aytiladi. Chunki ,bularda markaz ham, radius ham bilinib turibdi. Qavslarni ochib, tartibga solsak,

$$x^2+y^2+px+qy+c=0 \qquad (4)$$

hosil bo’ladi. Bu esa, aylananing umumiy tenglamasi deb aytiladi.

II. <u>Ellips.</u> Fokus deb ataluvchi ikki nuqtagacha bo’lgan masofalari yig`indisi o’zgarmas bo’lgan nuqtalarning tekislikdagi geometrik o’rnini ellips deb ataymiz.



Jumla (ta`rifi)ni formulaga aylantirish uchun $F_1(-c;0)vaF_2(c;0)$ nuqtalarni fokus nuqtalari deb atab, M(x,y) nuqtani ellipsga taalluqli bolsin deb qaraymiz. Ta`rifga ko’ra

$F_1M + F_2M = const. F_1M = \sqrt{(x+c)^2 + y^2}\ \ F_2M = \sqrt{(x-c)^2 + y^2}$

Hozircha $const = 2a$ deb belgilab tursak, quyidagi hosil bo'ladi.

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

Agar tenglamani irratsionallikdan qutqarib,

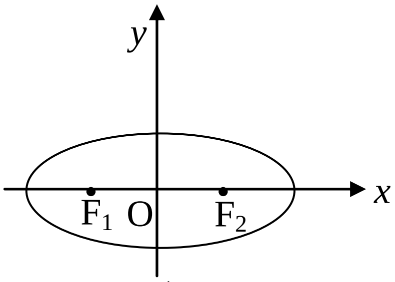
a^2-c^2 =b^2 deb belgilasak,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\ c^2 = a^2 - b^2\ \ (5)$$ ifodaga

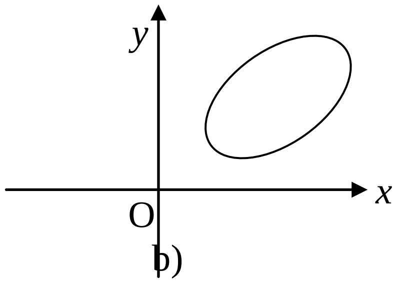
ega bo'lamiz .

$$\left.\begin{array}{l} Agar\, x = \pm a \ \text{ bo'lganda}, y = 0 \\ y = \pm b \ \text{ bo'lganda}, x = 0 \end{array}\right\}$$

kabi barcha cho'qqi nuqtalar holatini aniqlasak, (5) formula katta o'qi- 2a ga, kichik o'qi -2b ga va fokus oralig`i- 2c ga teng bo'lgan ellips tenglamasi ekanligiga iqror bo'lamiz. Grafikda koordinata o'qlariga simmetrik ellips vujudga kelganini ko'ramiz, chunki biz bu fokuslarni O*x* o'qiga va koordinata o'qiga simmetrik qilib tanlaganmiz.



a)



b)

Savol: (O'ylab ko'ring)

Keyngi shakldagi ellipsning tenglamasi qanaqa bo'ladi?

III. <u>Giperbola.</u> Fokus deb ataluvchi ikki nuqtagacha bo'lgan masofalari ayirmasining moduli o'zgarmas bo'lgan nuqtalarning tekislikdagi geometrik o'rniga giperbola deymiz.

F_1M-F_2M=const.

Yo'qoridagi II banddagi amallarni bajarsak,

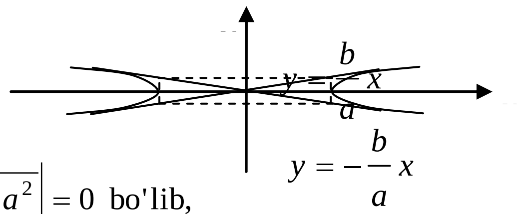
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (6)$$

hosil bo'ladi. Bu, fokuslari $F_1(-c;0), F_2(c;0)$ nuqtalarda, haqiqiy o'qi 2a va mavhum o'qi 2b bo'lgan giperbola tenglamasidir, bu yerda $c^2 - a^2 = b^2$.

Uning grafigini to'la tasavvur qilish, ya`ni aniq chizish uchun tenglamasini quyidagi shaklga keltiramiz: $y = \pm\frac{b}{a}\sqrt{x^2 - a^2}$ (7)

Ko'rinib turibdiki ,aniqlanish sohasi $x^2 - a^2 \ge 0 - dan kelib chiqadi (-\infty;-a) \cup (a;\infty)$.

$(-a;\ 0)\ va\ (a;\ 0)$ nuqtalar uning uchi bo'lib, grafigi simmetrik qanotlardan iborat bo'ladi.

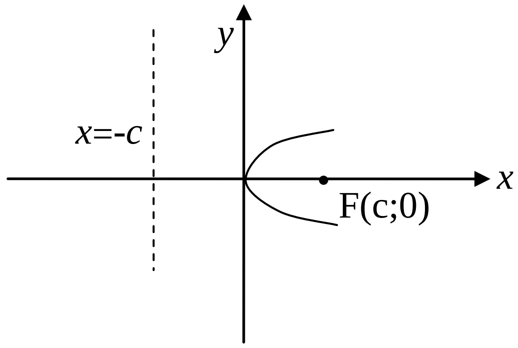


$y = \pm\frac{b}{a}x$ chiziq bilan solishtirsak $\lim\limits_{x\to\infty}\left|\frac{b}{a}x - \frac{b}{a}\sqrt{x^2 - a^2}\right| = 0$ bo'lib,

$grafigi \ \ y = \frac{b}{a} x \ va \ y = \frac{b}{a} x \quad (8)$

to'g`ri chiziq grafiklari orasida bo'ladi. $x \to \infty$ bilan (7) tenglama grafigi (8) tenglama grafigiga cheksiz yaqinlashib boradi, ammo unga xech qachon etib ololmaydi, chunki

$$x^2 - a^2 < x^2$$



Shu sababli (8) ifodalovchi chiziqqa asimptotik (etaklovchi-ergashtiruvchi) chiziqlar deb aytiladi.

IV. Parabola. Fokus deb ataluvchi F(c;0) va direktrisa deb ataluvchi x=c tug`ri chiziqdan barobar uzoqlikda joylashgan nuqtalarning tekislikdagi geometrik o'rniga parabola deb ataymiz.

DM=MF D(-c ,y)
DM=c+x D(x,y)
F (c,0)

$$MF = \sqrt{(x-c)^2 + y^2}$$

$$c^2 + 2cx + x^2 - 2cx + c^2 + y^2$$

$$y^2 = 4cx$$

Ko'pincha , hisoblarda $2c = p \ deb \left(c = \frac{p}{2} \right)$ olib tenglamani $y^2 = 2px$ (9)

ko'rinishda yozadi va parabolaning kanonik tenglamasi deb yuritiladi. Bu Ox bo'lgan egri chiziqdir.

Savol 1. Koordinat boshiga va o'qlarga simmetrik bo'lmaganda tenglamasi qanday yoziladi?

2. Uchi koordinat boshiga va o'qlarga simmetrik bo'lgan parabola tenglamasi qanday yoziladi?

J: $x^2 = 2py$ bo'ladi.

V. Nomlari zikr qilingan egri chiziqlarning shakllarining biridan ikkinchisiga o'tishini ifodalovchi kattalik aniqlangan va y ekstsentresitet ε - deb atalgan.

$$\varepsilon = \frac{c}{a} \quad Paraboladа \ \varepsilon = 1$$

$$Giperbolada \ \varepsilon > 1$$

$$Ellipsda \ \varepsilon < 1 \quad (10)$$

Ekstsentesitet orqali fokal radiuslarni topish formulasi qo'yidagicha

$$a)\left.\begin{array}{l} r_1 = a - \varepsilon x \\ r_2 = a + \varepsilon x \end{array}\right\} \text{ellipsuchun}$$

$$b)\left.\begin{array}{l} r_1 = \varepsilon x - a \\ r_2 = \varepsilon x + a \\ x > 0 \end{array}\right\} \text{va} \quad \left.\begin{array}{l} r_1 = a - \varepsilon x \\ r_2 = -a - \varepsilon x \\ x < 0 \end{array}\right\} \text{giperbola uchun}$$

$$r = x + \frac{p}{2}$$

$$\varepsilon = 1 \quad - \text{parabola uchun}$$

VI. Ikkinchi tartibli egri chiziqlar umumiy tenglamasining xususiy xol shart (belgi) lari.

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \qquad (1.a)$$

a)Agar A=C,B=0 bolganda (1.a) tenglama aylanani beradi.

б)$AC-B^2 > 0$ ellipsni berai.

в) $AC-B^2 < 0$ giperbolani beradi.

г) $AC-B^2 = 0$ o'zaro kesishuvchi yoki o'zaro parallel yoki mavxum egri chiziqlar yoki parabolani beradi. Xususiy parabola bolganda

$$\left.\begin{array}{l} A = 0 \\ B = 0 \end{array}\right\} \text{yoki} \quad \left.\begin{array}{l} B = 0 \\ C = 0 \end{array}\right\} \text{bo'lishi shart}$$

VII. Chiziqning to'gri aniq bo'ldi deylik. Endi

$$\left.\begin{array}{l} x = u + a, \\ y = v + b \end{array}\right\} \qquad (12)$$

almashtirish orqali uO_1v - koordinatalar sistemasiga ko'chamiz. Keyin

$$\left.\begin{array}{l} u - t\cos a - T\sin a, \\ v = t\sin a + T\cos a \end{array}\right\} \qquad (13)$$

almashtirish orqali uO_1v -sistemani α burchakka burib yangi tO_1T sistemaga o'tamiz. Natijada tO_1T sistemadagi tenglamaga ega bo'lamiz, ya`ni

$$\lambda_1 u^2 + \lambda_2 v^2 + 2D'u + 2E'v + F' = 0 \ \text{ifodaga ega bo'lamiz.}$$

$$\text{Bunda} \left.\begin{array}{l} D' = A\cos\alpha + E\sin\alpha \\ E' = -D\sin\alpha + E\cos\alpha \\ F' = F \\ tg\alpha = \dfrac{\lambda - A}{B} = \dfrac{A}{\lambda - c} \end{array}\right\} \qquad (14)$$

Misol 1. $5x^2+8xy+5y^2-18x-18y+9=0$

A=C=5, B=4, D=-9,. E=-9, F=9.

$$\begin{vmatrix} A-\lambda & B \\ B & C-\lambda \end{vmatrix} = \begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix} = 0 \ \lambda^2 - 10\lambda + 9 = 0, \quad \begin{array}{l} \lambda_1 = 9, \\ \lambda_2 = 1 \end{array}$$

$$tg\alpha = \frac{\lambda - A}{B} = \frac{9-5}{5} = \frac{4}{5} \Rightarrow \sin\alpha = \frac{4}{\sqrt{41}}; \cos\alpha = \frac{5}{\sqrt{41}}$$

$$D' = 5\cdot\frac{5}{\sqrt{41}} + (-9)\cdot\frac{4}{\sqrt{41}} \approx -9\sqrt{2}; \ E' = 0; F' = 9.$$

Demak, tenglamamiz $9u^2 + v^2 - 18\sqrt{2u+9} = 0$ ko'rinishni oladi. Bundan

$$\frac{\left(u-\sqrt{2}\right)^2}{1} + \frac{v^2}{9} = 1 \qquad \left.\begin{matrix} u-\sqrt{2} = t \\ v = T \end{matrix}\right\} almahtirish \ olsak, \ ellipsning \ \frac{t^2}{1} + \frac{T^2}{9} = 1$$

kanonik tenglamasi xosil bo'ladi.